THEOREM: THE CONSTANT RULE

The derivative of a constant function is zero. That is, if c is a real number,

then

$$\frac{d}{dx}[c] = 0$$

Example 1: Find the derivative of the function g(x) = 5.

$\frac{dx}{g'(x) = 0}$

THEOREM: THE POWER RULE

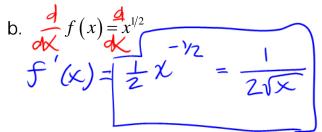
If *n* is a rational number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx} \left[x^n \right] = nx^{n-1}$$

For f to be differentiable at x = 0, n must be a number such that x^{n-1} is defined on an interval containing zero.

Example 2: Find the following derivatives.

a.
$$f(x) = -5\chi^{-6} = -\frac{5}{\chi^{6}}$$

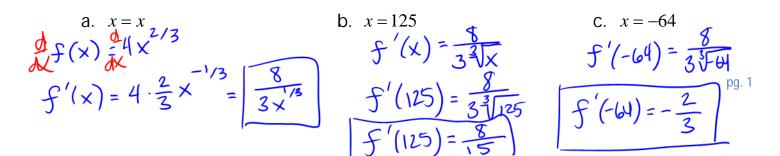


THEOREM: THE CONSTANT MULTIPLE RULE

If f is a differentiable function and c is a real number, then cf is also differentiable and

$$\frac{d}{dx}\left[cf\left(x\right)\right] = cf'(x)$$

Example 3: Find the slope of the graph of $f(x) = 4x^{2/3}$ at

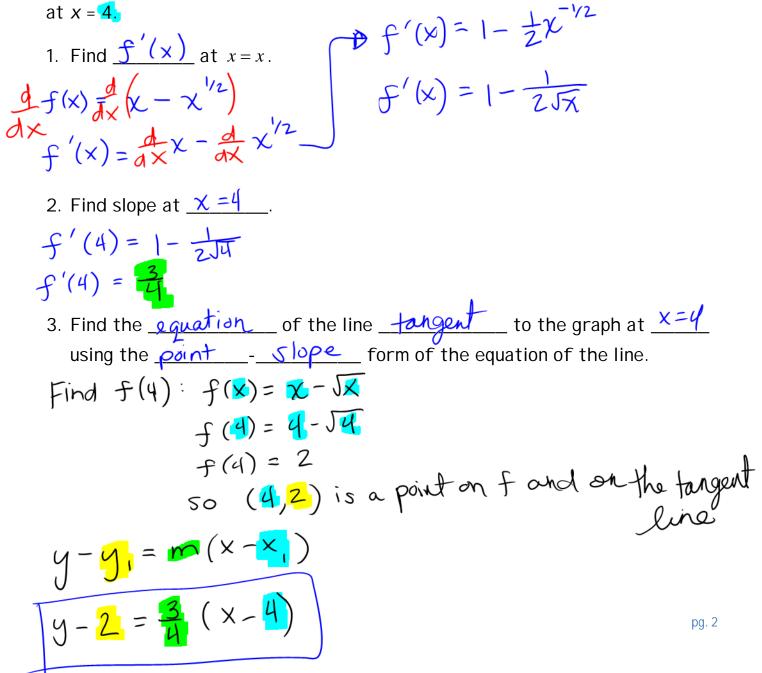


THEOREM: THE SUM AND DIFFERENCE RULES

The sum (or difference) of two differentiable functions f and g is itself differentiable. Moreover, the derivative of f + g (or f - g) is the sum (or difference) of the derivatives of f and g.

$$\frac{d}{dx} \left[f(x) + g(x) \right] = f'(x) + g'(x)$$
$$\frac{d}{dx} \left[f(x) - g(x) \right] = f'(x) - g'(x)$$

Example 4: Find the equation of the line tangent to the graph of $f(x) = x - \sqrt{x}$



THEOREM: DERIVATIVES OF THE TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}[\sin x] = \cos x \qquad \qquad \frac{d}{dx}[\cos x] = -\sin x$$
$$\frac{d}{dx}[\csc x] = -\csc x \cot x \qquad \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$
$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

Example 5: Find the derivative of the following functions:

$$a_{x}^{\dagger} f(x) = \frac{\sin x}{6} \frac{1}{x} \int \sin x$$

$$f'(x) = \frac{1}{6} \cos x$$

$$b_{x}^{\dagger} r(\theta) \frac{1}{6} 5\theta - 3\cos \theta$$

$$r'(\theta) = 5 - 3(-\sin \theta)$$

$$r'(\theta) = 5 - 3(-\sin \theta)$$

$$r'(\theta) = 5 + 3\sin \theta$$

$$c. h(t) = \frac{\cos t}{\cot t} = \frac{\cos t}{\cos t} = \cot t + \frac{\sin t}{\cos t} = \sin t$$

$$d_{x} h(t) \frac{1}{4t} \sin t + \frac{\sin t}{1} + \frac{\ln^{2}(t)}{\ln^{2}(t)} = \cos t$$

$$g_{x}^{\dagger} f(x) = 0 + 3 \sec t - \frac{\sin t}{\cos t}$$

$$f'(x) = -3 \sec t - \frac{\sin t}{\cos t} = \sin t$$

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THEOREM: THE PRODUCT RULE

The product of two differentiable functions f and g is itself differentiable. Moreover, the derivative of fg is the derivative of the first function times the second function, plus the first function times the derivative of the second function.

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

This rule extends to cover products of more than two factors. For example the derivative of the product of functions fghk is

 $\frac{d}{dx}[fghk] = f'(x)g(x)h(x)k(x) + f(x)g'(x)h(x)k(x) + f(x)g(x)h'(x)k(x) + f(x)g(x)h(x)k'(x)$

Example 6: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

a.
$$g(x) = x \cos x$$

 $dx \quad dx$
 $g'(x) = \left(\frac{d}{dx}x\right) \cos x + x \left(\frac{d}{dx}\cos x\right)$
 $g'(x) = 1 \cos x + x (-\sin x)$
 $g'(x) = \cos x - x \sin x$

$$b_{1t}^{d} h(t) = (3 - \sqrt{t})^{2} = \frac{d}{dt} (3 - t^{1/2})(3 - t^{1/2})]$$

$$h'(t) = \begin{bmatrix} d & (3 - t^{1/2}) \\ d & (3 - t^{1/2}) + (3 - t^{1/2}) \\ d & (3 - t^{1/2}) + (3 - t^{1/2}) \end{bmatrix}$$

$$h'(t) = (-\frac{1}{2}t^{-1/2})(3 - t^{1/2}) + (3 - t^{1/2})(-\frac{1}{2}t^{-1/2})$$

$$h'(t) = -t^{-1/2} (3 - t^{1/2}) + (3 - t^{1/2})(-\frac{1}{2}t^{-1/2})$$

$$h'(t) = -(3 - t^{1/2}) + (3 - t^{1/2}) + (3 - t^{1/2}) + (3 - t^{1/2})(-\frac{1}{2}t^{-1/2}) + (3 - t^{1/2})(-\frac{1}{2}t^{-1/2})(-\frac{1}{2}t^{-1/2})(-\frac{1}{2}t^{-1/2}) + (3 - t^{1/2})(-\frac{1}{2}t^{-1/2})(-\frac{1$$

THEOREM: THE QUOTIENT RULE

The quotient of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$. Moreover, the derivative of f/g is the derivative of the numerator times the denominator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Example 7: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

$$a_{N}^{*}g(x) \stackrel{d}{=} \frac{2x}{5x^{2}+3}$$

$$g'(x) = \frac{(1-x)(5x^{2}+3)-(2x)(4x)(5x^{2}+3)]}{(5x^{2}+3)^{2}}$$

$$g'(x) = \frac{2(5x^{2}+3)-2x(10x)}{(5x^{2}+3)^{2}}$$
Simplifying LTS

$$b_{N}^{*}h(t) \stackrel{d}{=} \frac{t}{\sqrt{t-1}}$$

$$b_{N}^{*}(t) = \frac{(4x)(t^{1/n}-1)-t(4x)(t^{1/n}-1)-t(4x)(t^{1/n}-1))}{(4x-1)-t(4x)(t^{1/n}-1)-t(4x)(t^{1/n}-1))}$$

$$h'(t) = \frac{t^{1/n}-1-\frac{1}{2}t^{1/2}}{(4x-1)^{2}}$$

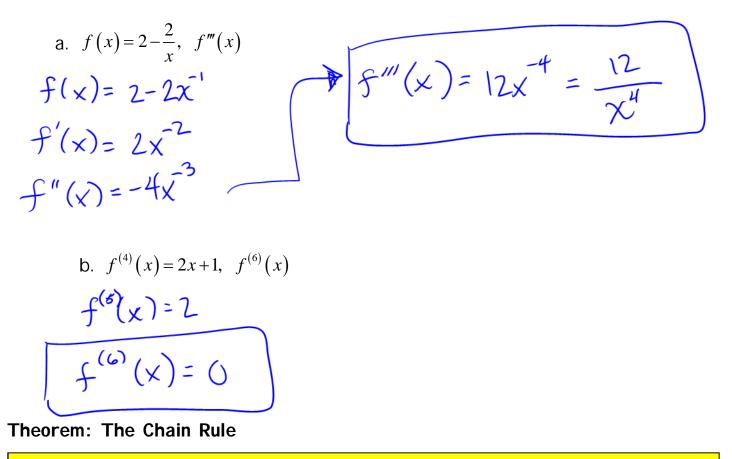
$$h'(t) = \frac{1(t^{1/n}-1)-t(4x)(t^{1/n}-1)}{(4x-1)^{2}}$$

$$h'(t) = \frac{2t^{1/2}-1}{(4x-1)^{2}}$$

$$h'(t) = \frac{2t^{1/2}-1}{(4x-1)^{2}}$$

$$h'(t) = \frac{t^{1/n}-2}{2(4x-1)^{2}}$$

Example 8: Find the given higher-order derivative.



If y = f(u) is a differentiable function of u and u = g(x) is a differentiable function of x, then y = f(g(x)) is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \text{ or } \frac{d}{dx} [f(g(x))] = f'(g(x))g'(x).$

Example 9: Find the derivative using the Chain Rule.

a.
$$y = (\sqrt{x} - x^2)^{10}$$

b. $f(x) = \frac{1}{\sqrt{2-x}} = (2-x)^{-1/2}$
f'(x) = $-\frac{1}{2}(2-x)^{-\frac{3}{2}} \frac{1}{\sqrt{2-x}}$
f'(x) = $-\frac{1}{2}(2-x)^{-\frac{3}{2}} \frac{1}{\sqrt{2-x}}$

Example 10: Find the derivative of the following functions.

a.
$$y = \sin x$$

 $y' = \cos x$
b. $y = \sin 2x$
 $y' = \frac{\cos(2x)(\frac{1}{4x}, 2x)}{(\frac{1}{6}, \frac{1}{2}, 2\cos(2x))(\frac{1}{4x}, 2x)}$
 $y' = \frac{\cos(x^{2})(\frac{1}{4x}, 2x)}{(\frac{1}{6}, \frac{1}{6}, \frac{1}{6},$

g.
$$h(x) = x \sin^2 4x = x (\sin 4x)^2$$

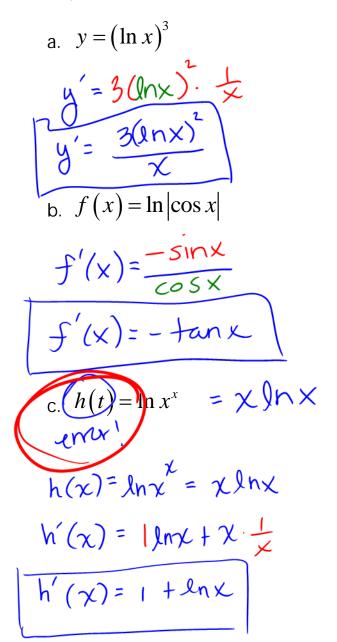
 $h'(x) = \frac{(\sin 4x)^2 + x (\sin 4x)(\cos 4x)(4)}{(x)^2 + \sin^2 4x + 8x \sin 4x \cos 4x} = \sin^2 4x + 4x \sin 8x$

Theorem: Derivative of the Natural Logarithmic Function

Let *u* be a differentiable function of *x*.

1.
$$\frac{d}{dx} [\ln x] = \frac{1}{x}$$
, $\chi > O$

Example 11: Find the derivative.



2. $\frac{d}{dx}[\ln u] = \frac{u'}{u}, \quad u > 0$ $\frac{d}{dx}[\ln |u|] = \frac{u'}{u}, \quad (-\infty, 0)(u)$ $\frac{d}{dx}[\ln |u|] = \frac{u'}{u}, \quad (-\infty, 0)(u)$

Theorem: Derivative of the Natural Exponential Function

Let *u* be a differentiable function of *x*. 1. $\frac{d}{dx} \left[e^x \right] = e^x$ 2. $\frac{d}{dx} \left[e^u \right] = e^u u'$

Example 12: Find the derivative.

a.
$$y = xe^{-x}$$

 $y' = |e^{-x} + xe^{-x}(-1)$
 $y' = e^{-x}(1-x)$

b.
$$f(x) = e^{\sin 2x}$$

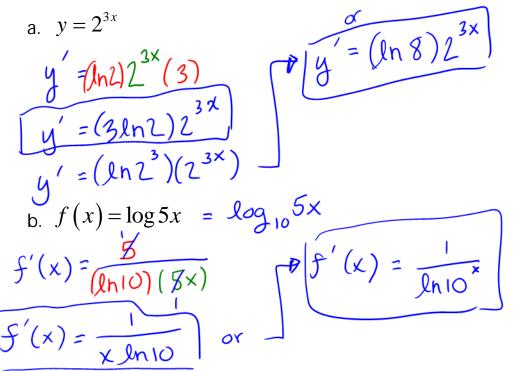
 $f'(x) = e^{\sin 2x}(\cos 2x) \cdot 2$
 $f'(x) = 2(\cos 2x)e^{\sin 2x}$
c. $h(t) = \frac{e^{t}}{\ln e^{\sqrt{t}}} = \frac{e^{t}}{\sqrt{t}} = \frac{e^{t}}{\sqrt{t}} = \frac{e^{t}}{(t(t))} = \frac{e^{t}}{t'^{n}}$
 $h'(t) = \frac{e^{t}t'^{n} - e^{t}(\frac{1}{2}t'^{n})}{(t'^{n})^{2}}$
 $h'(t) = \frac{1}{2}t^{-1}e^{t}(t'-1)$
 $\frac{t}{h'(t)} = \frac{e^{t}(t-1)}{2t^{-1}}$

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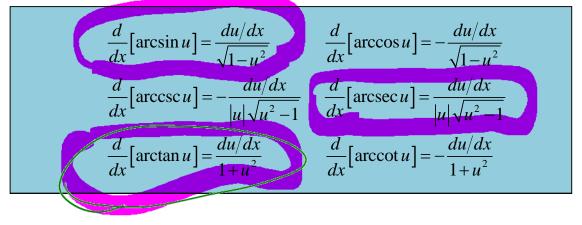
Theorem: Derivatives for Bases other than e

Let *a* be a positive real number
$$(a \neq 1)$$
 and let *u* be a differentiable function of *x*.
1. $\frac{d}{dx} \left[a^x \right] = (\ln a) a^x$
2. $\frac{d}{dx} \left[a^u \right] = (\ln a) a^u u'$
3. $\frac{d}{dx} \left[\log_a x \right] = \frac{1}{(\ln a)x}$
4. $\frac{d}{dx} \left[\log_a u \right] = \frac{u'}{(\ln a)u}$

Example 13: Find the derivative.



THEOREM: DERIVATIVES OF THE INVERSE TRIGONOMETRIC FUNCTIONS (u is a function of x)



Example 14: Find the derivative.

a. $y = \arctan 3x - \ln(1+9x^2)$ $y' = \frac{3}{1+(3x)^2} - \frac{18x}{1+9x^2}$ $y' = \frac{3-18x}{1+9x^2}$

b.
$$f(x) = x \arcsin \sqrt{x}$$

 $f'(x) = | \arctan x + x \frac{\frac{1}{2}x'^2}{1 - (\sqrt{x})^2}$
 $f'(x) = \arcsin x + \frac{x}{2\sqrt{x}(1 - x)}$
 $f'(x) = \frac{2\sqrt{x}(1 - x)}{2\sqrt{x}(1 - x)}$